

**Paper 3C/4C: Further Mechanics 1 Mark Scheme**

Question	Scheme	Marks	AOs
1	Use Impulse-momentum principle	M1	2.1
	$2\mathbf{i} - \mathbf{j} = 0.5\mathbf{v} - 0.5(4\mathbf{i} + \mathbf{j})$	A1	1.1b
	$\frac{1}{2}\mathbf{v} = 4\mathbf{i} - \frac{1}{2}\mathbf{j}, \quad \mathbf{v} = 8\mathbf{i} - \mathbf{j} \text{ (m s}^{-1}\text{)}$	A1	1.1b
	Use of $\text{KE} = \frac{1}{2}m \mathbf{v} ^2 - \frac{1}{2}m \mathbf{u} ^2$	M1	2.1
	$= \frac{1}{2} \times 0.5 \times \{(64 + 1) - (16 + 1)\}$	A1	1.1b
	$= \frac{1}{4} \times 48 = 12 \text{ (J)} \quad *$	A1*	1.1b
		(6)	
<b>(6 marks)</b>			
<b>Notes:</b>			
<p><b>M1:</b> Difference of terms &amp; dimensionally correct</p> <p><b>A1:</b> Correct unsimplified equation</p> <p><b>A1:</b> cao</p> <p><b>M1:</b> Must be a difference of two terms Must be dimensionally correct</p> <p><b>A1:</b> Correct unsimplified equation</p> <p><b>A1*:</b> Complete justification of given answer</p>			

Question	Scheme	Marks	AOs
<b>2(a)</b>	$R = 5g \cos \alpha \left( = 5g \times \frac{4\sqrt{3}}{7} = 48.497\dots \right)$	M1	3.4
	Force due to friction = $\mu \times 5g \cos \alpha$	M1	3.4
	Work-Energy equation	M1	3.4
	$\frac{1}{2} \times 5 \times 64 = 5 \times 9.8 \times 14 \sin \alpha + 14\mu R$	A1	1.1b
	$\mu = 0.0913$ or $0.091$	A1	1.1b
		<b>(5)</b>	
<b>(b)</b>	Appropriate refinement	B1	3.5c
		<b>(1)</b>	
<b>(6 marks)</b>			
<b>Notes:</b>			
<p><b>(a)</b>  <b>M1:</b> Condone sin/cos confusion  <b>M1:</b> Use of <math>\mu \times</math> their R  <b>M1:</b> Must be using work-energy. Requires all terms  Condone sin/cos confusion, sign errors and their R  <b>A1:</b> Correct in <math>\theta</math> and <math>\mu R</math>  <b>A1:</b> Accept 0.0913 or 0.091</p>			
<p><b>(b)</b>  <b>B1:</b> e.g.  - do not model the parcel as a particle and therefore take air resistance into account  - take into account the dimensions/uniformity of the parcel</p>			

Question	Scheme	Marks	AOs
<b>3(a)</b>	Use NEL to find the speed of particle after the first impact $= eu = \frac{3}{4}u \frac{\pi}{2}$	B1	3.4
	Impulse = $\lambda mu = mv - mu = \pm \left[ \frac{3}{4}mu - (-mu) \right]$	M1	3.1b
	$\lambda = \frac{7}{4}$	A1	1.1b
		<b>(3)</b>	
<b>(b)</b>	Use NEL to find the speed of the particle after the second impact $= \frac{3}{4} \times \frac{3}{4}u = \frac{9}{16}u$	B1	3.4
	Use of $s = vt$ to find total time	M1	3.1b
	$7 = \frac{2}{u} + \frac{4}{\frac{3}{4}u} + \frac{2}{\frac{9}{16}u} \left( = \frac{2}{u} + \frac{16}{3u} + \frac{32}{9u} \right)$	A1	1.1b
	Solve for $u$ : $63u = 18 + 48 + 32$	M1	1.1b
	$u = \frac{98}{63} = \frac{14}{9} (= 1.5)$	A1	1.1b
		<b>(5)</b>	
<b>(8 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>B1:</b> Using Newton's experimental law as a model to find the speed after the first impact			
<b>M1:</b> Must be a difference of two terms, taking account of the change in direction of motion			
<b>A1:</b> cao			
<b>(b)</b>			
<b>B1:</b> Using NEL as a model to find the speed after the second impact			
<b>M1:</b> Needs to be used for at least one stage of the journey			
<b>A1:</b> Ur equivalent			
<b>M1:</b> Solve their linear equation for $u$			
<b>A1:</b> Accept 1.56 or better			

Question	Scheme	Marks	AOs
<b>4(a)</b>	Complete strategy to find the kinetic energy after the second impact	M1	3.1b
	Parallel to $AB$ after collision: $u \cos 60^\circ$	M1	3.1b
	Perpendicular to $AB$ after collision: $\frac{1}{\sqrt{3}}u \sin 60^\circ$	M1	3.4
	Components of velocity after first impact: $\frac{u}{2}, \frac{u}{2}$	A1	1.1b
	Parallel to $BC$ after collision: $\frac{u}{2} \left( u \times \frac{1}{\sqrt{3}} \sin 60^\circ \right)$	M1	3.1b
	Perpendicular to $BC$ after collision: $\sqrt{\frac{2}{5}} \times \frac{u}{2} \left( = \frac{1}{\sqrt{10}}u \right)$ $\left( \sqrt{\frac{2}{5}} \times u \cos 60^\circ \right)$	M1	3.4
	Components of velocity after second impact: $\frac{u}{2}, \frac{u}{\sqrt{10}}$	A1	1.1b
	Final KE = $\frac{1}{2}m \left( \frac{u^2}{4} + \frac{u^2}{10} \right) \left( = \frac{mu^2}{2} \times \frac{7}{20} \right)$		
	Fraction of initial KE = $\frac{\frac{mu^2}{2} \times \frac{7}{20}}{\frac{mu^2}{2}} = \frac{7}{20} = 35\% *$	A1*	2.2a
	<b>(8)</b>		
<b>(b)</b>	The answer is too large - rough surface means resistance so final speed will be lower	B1	3.5a
		<b>(1)</b>	
<b>(9 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>M1:</b> Use of CLM parallel to the wall. Condone sin/cos confusion			
<b>M1:</b> Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion			
<b>A1:</b> Both components correct with trig substituted (seen or implied)			
<b>M1:</b> Use of CLM parallel to the wall. Condone sin/cos confusion			
<b>M1:</b> Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion			
<b>A1:</b> Both components correct with trig substituted (seen or implied)			
<b>M1:</b> Correct expression for total KE using their components after 2nd collision			
<b>A1*:</b> Obtain <b>given answer</b> with sufficient working to justify it			
<b>(b)</b>			
<b>B1:</b> Clear explanation of how the modelling assumption has affected the outcome			

Question	Scheme	Marks	AOs
<b>5(a)</b>	Use of $P = Fv$ : $F = \frac{12000}{20}$	B1	3.3
	Equation of motion: $F - (200 + 2v) = 600a$	M1	3.4
	$600 - 240 = 600a$	A1ft	1.1b
	$360 = 600a, a = 0.6 \text{ (m s}^{-2}\text{)}$	A1	1.1b
		<b>(4)</b>	
<b>(b)</b>	Equation of motion:	M1	3.3
	$\frac{12000}{w} - (200 + 2w) - 600g \sin \theta = -600 \times 0.05$	A1	1.1b
		A1	1.1b
	3 term quadratic and solve: $2w^2 + 590w - 12000 = 0$	M1	1.1b
	$w = \frac{-590 + \sqrt{590^2 + 96000}}{4} = 19.1 \text{ (m s}^{-1}\text{)}$	A1	1.1b
	<b>(5)</b>		
<b>(9 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>B1:</b> 600 or equivalent			
<b>M1:</b> Use the model to form the equation of motion Must include all terms .Condone sign errors			
<b>A1ft:</b> Correct for their $F$			
<b>A1:</b> cao			
<b>(b)</b>			
<b>M1:</b> Use the model to form the equation of motion All terms needed. Condone sign errors and sin/cos confusion			
<b>A1:</b> All correct A1A1 One error A1A0			
<b>M1:</b> Dependent on the preceding M1. Use the equation of motion to form a 3-term quadratic in $w$ only			
<b>A1:</b> Accept 19. Do not accept more than 3 s.f.			

Question	Scheme	Marks	AOs
<b>6(a)</b>			
	Overall strategy to find $\mathbf{V}_A$	M1	3.1a
	Velocity of A perpendicular to loc after collision = $3\mathbf{j}$ ( $\text{m s}^{-1}$ )	B1	3.4
	CLM parallel to loc	M1	3.1a
	$2m \times 3 - 3m \times 5 = 3mw - 2mv$ ( $-9 = 3w - 2v$ )	A1	1.1b
	Correct use of impact law	M1	3.1a
	$v + w = \frac{1}{4}(3 + 5)$ ( $= 2$ )	A1	1.1b
	Solve for $w$ $3w - 2v = -9$ $2v + 2w = 4$		
	$\mathbf{v}_B = -\mathbf{i} + 2\mathbf{j}$ ( $\text{m s}^{-1}$ ),	A1ft	1.1b
		(7)	
<b>(b)</b>	$\cos \theta = \frac{(-5\mathbf{i} + 2\mathbf{j}) \cdot (-\mathbf{i} + 2\mathbf{j})}{\sqrt{29}\sqrt{5}}$	M1	3.1a
	$\theta = 41.63\dots^\circ = 42^\circ$ (nearest degree)	A1	1.1b
	Alternative method: $\tan^{-1} 2 - \tan^{-1} \frac{2}{5} = 41.63\dots^\circ = 42^\circ$ (nearest degree)		
		(2)	
<b>(9 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>M1:</b> Correct overall strategy to form sufficient equations and solve for $\mathbf{V}_A$			
<b>B1:</b> Use the model to find the component of $\mathbf{V}_A$ perpendicular to the line of centres			
<b>M1:</b> Use CLM to form equation in $v$ and $w$ . Need all 4 terms, dimensionally correct			
<b>A1:</b> Correct unsimplified			
<b>M1:</b> Must be used the right way round			
<b>A1:</b> Correct unsimplified			
<b>A1ft:</b> $\mathbf{v}_B$ correct. Follow their $2\mathbf{j}$			
<b>(b)</b>			
<b>M1:</b> Complete method for finding the required angle. Follow their $\mathbf{v}_B$			
<b>A1:</b> cao			

Question	Scheme	Marks	AOs
<b>7(a)</b>	In equilibrium $\Rightarrow$ no resultant vertical force	M1	2.1
	$\frac{3mgx}{a} = mg$	A1	1.1b
	$x = \frac{a}{3}, \quad d = \frac{4}{3}a \quad *$	A1*	2.2a
		<b>(3)</b>	
<b>(b)</b>	Equation of motion:	M1	3.1a
	$\frac{3mga}{a} - mg = m\ddot{x}$	A1	1.1b
	$\ddot{x} = 2g$	A1	1.1b
		<b>(3)</b>	
<b>(c)</b>	Max speed at equilibrium position	B1	3.1a
	Work energy & use of EPE = $\frac{\lambda x^2}{2a}$	M1	3.1a
	$\frac{3mga^2}{2a} = \frac{3mg\left(\frac{a}{3}\right)^2}{2a} + \frac{1}{2}mv^2 + mg\frac{2a}{3}$	A1 A1	1.1b 1.1b
	$\frac{1}{2}v^2 = ga\left(\frac{3}{2} - \frac{1}{6} - \frac{2}{3}\right) = \frac{2}{3}ga, \quad v = \sqrt{\frac{4ga}{3}}$	A1	1.1b
		<b>(5)</b>	
<b>(d)</b>	At max ht. KE = 0. EPE lost = GPE gained	M1	3.1a
	$\frac{3mga^2}{2a} = mgh$	A1	1.1b
	$OB = \frac{a}{2}$	A1	1.1b
		<b>(3)</b>	
<b>(14 marks)</b>			

**Question 7 notes:****(a)****M1:** Use  $T = \frac{\lambda x}{a}$  to form equation for equilibrium**A1:** Correct unsimplified equation**A1\*:** Requires sufficient working to justify given answer plus a 'statement' that the required result has been achieved**(b)****M1:** Use  $T = \frac{\lambda x}{a}$  to form equation of motion

Need all 3 terms. Condone sign errors

**A1:** Correct unsimplified equation**A1:** cao**(c)****B1:** Seen or implied**M1:** Form work-energy equation. All 4 terms needed  
Condone sign errors**A1:** Correct unsimplified equation A1A1  
One error in the equation A1A0**A1:** cao**(d)****M1:** Form energy equation**A1:** Correct unsimplified equation**A1:** cao

Question	Scheme	Marks	AOs
<b>8(a)</b>			
	Complete overall strategy to find $v$	M1	3.1a
	Use of CLM	M1	3.1a
	$2m \times 2u - 5m \times u = 5m \times v - 2m \times w$ , ( $-u = 5v - 2w$ )	A1	1.1b
	Use of Impact law:	M1	3.1a
	$v + w = e(2u + u)$	A1	1.1b
	Solve for $v$ : $-u = 5v - 2w$ $6eu = 2v + 2w$		
	$7v = u(6e - 1)$ ( $v = \frac{u}{7}(6e - 1)$ )	A1	1.1b
	Direction of $Q$ reversed: $v > 0$	M1	3.4
	$\Rightarrow 1 \geq e > \frac{1}{6}$	A1	1.1b
		<b>(8)</b>	
<b>(b)</b>	$e = \frac{1}{3} \Rightarrow v = \frac{u}{7}, w = \frac{6u}{7}$	B1	2.1
	Equation for KE lost	M1	2.1
	$\frac{1}{2} \times 2m \left( 4u^2 - \frac{36u^2}{49} \right) + \frac{1}{2} \times 5m \left( u^2 - \frac{u^2}{49} \right)$	A1 A1	1.1b 1.1b
	$\frac{1}{2} mu^2 \left( 8 - \frac{72}{49} + 5 - \frac{5}{49} \right) = \frac{40mu^2}{7}$ *	A1*	2.2a
		<b>(5)</b>	
<b>(c)</b>	Increase $e \Rightarrow$ more elastic $\Rightarrow$ less energy lost	B1	2.2a
		<b>(1)</b>	
<b>(14 marks)</b>			

**Question 8 notes:****(a)****M1:** Complete strategy to form sufficient equations in  $v$  and  $w$  and solve for  $v$ **M1:** Use CLM to form equation in  $v$  and  $w$ 

Needs all 4 terms &amp; dimensionally correct

**A1:** Correct unsimplified equation**M1:** Use NEL as a model to form a second equation in  $v$  and  $w$ . Must be used the right way round**A1:** Correct unsimplified equation**A1:** for  $v$  or  $7v$  correct**M1:** Use the model to form a correct inequality for their  $v$ **A1:** Both limits required**(b)****B1:** Or equivalent statements**M1:** Terms of correct structure combined correctly**A1:** Fully correct unsimplified A1A1

One error on unsimplified expression A1A0

**A1\*:** cso. plus a 'statement' that the required result has been achieved**(c)****B1:** "less energy lost" or equivalent